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Shape of Pion Distribution Amplitude

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A scenario is investigated in which the leading-twist pion distribution amplitude $\varphi_\pi(x)$ is approximated by the pion decay constant f_π for all essential values of the light-cone fraction x. A model for the light-front wave function $\Psi(x, k_\perp)$ is proposed that produces such a distribution amplitude and has a rapidly decreasing (exponential for definiteness) dependence on the light-front energy combination $k_\perp^2/x(1-x)$. It is shown that this model easily reproduces the fit of recent large-Q^2 BaBar data on the photon-pion transition form factor. Some aspects of scenario with flat pion distribution amplitude are discussed.

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I. INTRODUCTION

The pion distribution amplitude (DA) $\varphi_\pi(x)$ [1, 2] is an important function accumulating information about momentum sharing between the quarks of the pion when the latter is in its valence $\bar{q}q$ configuration. It is an inherent element of perturbative QCD calculations of hard exclusive reactions involving the pion. From the solution [3, 4], [2] of the evolution equation for the pion DA it follows that independently of its shape at low normalization point $\mu_0 \lesssim 1\,\text{GeV}$, at large values of the probing momentum the pion DA acquires universal asymptotic form: $\varphi_\pi(x, \mu \to \infty) \to 6 f_\pi x(1-x)$ [5]. However, in practical calculations, it is very important to know what is the shape of the pion DA at moderate and low scales μ. The standard measure of the width of the pion DA is the value $\langle \xi^2 \rangle$ of its second moment with respect to the relative momentum fraction variable $\xi = x - (1-x)$. QCD sum rule calculations [6] give large $\langle \xi^2 \rangle > 0.4$ values for this moment (compared to $\langle \xi^2 \rangle = 0.2$ for the asymptotic DA) which indicates that the pion DA is a wide function for $\mu^2 < 1\,\text{GeV}^2$. Recent lattice calculations [7, 8] give $\langle \xi^2 \rangle \gtrsim 0.3$ for μ^2 values in this region. A direct calculation of the pion DA in the Nambu-Jona-Lasinio model [9] (see also Ref. [10]) produces the result that $\varphi_\pi(x) = f_\pi$ for all values of the momentum fraction x, i.e., that pion DA is constant. The same result was obtained in the "spectral" quark model [11]. The value of $\langle \xi^2 \rangle$ for this "flat" DA is 1/3, which is compatible with the results of the lattice estimates, though smaller than the result of QCD sum rules. It should be noted that the usual procedure of reconstructing pion DA from its moments in the CZ approach (followed by essentially all other groups) is to build it as a sum of the lowest (two or three) Gegenbauer polynomials corresponding to multiplicatively renormalizable components of the pion DA evolution decomposition. Since these components have $x(1-x)$ as an overall factor, such a procedure excludes flat DAs from possible models. However, this restriction on the pion DA model building is just an assumption. In the present paper, our goal is to analyze the photon-pion transition form factor in a scenario with flat pion DA. The curve for the form factor which we obtain is in a very good agreement with recent BaBar data [12], which is basically the main motivation for our investigation.

The paper is organized in the following way. In Section II, we give an overview of the basic facts about the pion distribution amplitude: its definition, evolution and results concerning its shape. Section III is devoted to the study of the photon-pion transition form factor. We briefly describe pQCD results for this form factor, and then calculate it within the light-front formalism using a model wave function $\Psi(x, k_\perp)$ that reproduces flat pion DA after integration over quark transverse momentum $k_\perp$ and rapidly (exponentially) decreases for large values of the standard light-front energy combination $k_\perp^2/x(1-x)$. The $k_\perp$ width parameter σ of this wave function can be easily adjusted to produce a curve practically coinciding with the data fitting curve given in Ref. [12]. This value of σ corresponds to the value $\langle k_\perp^2 \rangle = (420\,\text{MeV})^2$ for the average transverse momentum squared, which has the magnitude that one would expect for the valence $\bar{q}q$ Fock component of the pion light-front wave function. We analyze the structure of the one-loop corrections for a flat DA in pQCD, and find out that the optimal value $\mu^2 = aQ^2$ of the normalization scale for a flat DA is very small. We argue that this is an evidence that the flat pion DA should not be evolved in our calculation of the photon-pion transition form factor. Finally, we discuss some aspects of the flat pion DA scenario and then we summarize the paper.

II. PION DISTRIBUTION AMPLITUDE: BASICS

A. Definition and Evolution

The pion distribution amplitude $\varphi_\pi(x)$ may be introduced [1] as a function whose x^n moments

$$f_n = \int_0^1 x^n \, \varphi_\pi(x) \, dx \tag{1}$$

are given by reduced matrix elements of twist-2 local operators

$$i^{n+1} \left\langle 0 | \bar{d}(0) \gamma_5 \left\{ \gamma_\nu D_{\nu_1} \ldots D_{\nu_n} \right\} u(0) | \pi^+, P \right\rangle = \left\{ P_\nu P_{\nu_1} \ldots P_{\nu_n} \right\} \, f_n \ , \tag{2}$$

or [2] as the $k_\perp$-integral

$$\varphi_\pi(x, \mu) = \frac{\sqrt{6}}{(2\pi)^3} \int_{k_\perp^2 \leq \mu^2} \Psi(x, k_\perp) \, d^2 k_\perp \tag{3}$$

of the light-front wave function $\Psi(x, k_\perp)$. The zeroth moment of $\varphi_\pi(x)$ corresponds to matrix element of the axial current, and is given by the pion decay constant f_π

$$\int_0^1 \varphi_\pi(x) \, dx = f_\pi \ , \tag{4}$$

which is known experimentally. In the conventions that we use, $f_\pi \approx 130\,\text{MeV}$. Eq.(4) gives an important constraint on the pion distribution amplitude (DA), fixing the integral under the $\varphi_\pi(x)$ curve, but it puts no restrictions on its shape. In fact, the pion DA depends on the renormalization scale μ that is used to define matrix elements of twist-2 local operators: $\varphi_\pi(x) \to \varphi_\pi(x, \mu)$. The evolution equation for the pion DA may be written either in matrix form [1]

$$\mu \frac{d}{d\mu} \, f_n(\mu) = \sum_{k=0}^{n} Z_{nk} f_k(\mu) \tag{5}$$

(see also [13]) or in kernel form [2]

$$\mu \frac{d}{d\mu} \varphi_\pi(x, \mu) = \int_0^1 V(x, y) \, \varphi_\pi(y, \mu) \, dy \ . \tag{6}$$

The solution of the evolution equation was obtained [3, 4], [2] in the form of expansion over Gegenbauer polynomials

$$\varphi_\pi(x, \mu) = 6 f_\pi \, x(1-x) \left\{ 1 + \sum_{n=1}^{\infty} a_{2n} C_{2n}^{3/2}(2x-1) \left[\ln(\mu^2/\Lambda^2) \right]^{-\gamma_{2n}/\beta_0} \right\} \ , \tag{7}$$

where $\gamma_{2n} > 0$ is the anomalous dimension of the composite operator with $2n$ derivatives, and β_0 is the lowest coefficient of the QCD β-function. As a result, when the normalization scale μ tends to infinity, the pion DA acquires a simple form [5]

$$\varphi_\pi(x, \mu \to \infty) = 6 f_\pi \, x(1-x) \ , \tag{8}$$

known as the "asymptotic DA".

B. Shape

The question, however, is what is the shape of the pion DA at low normalization scales $\mu \lesssim 1$ GeV. Some qualitative (and maybe overly simplistic by today's standards) argumentation about a possible shape of the pion DA was given in our 1980 papers [14, 15]. Namely, in case of a system of two equal-mass non-interacting particles, $\varphi(x) = \delta(x - 1/2)$. When the interaction is switched on, the DA broadens. The width Γ of $\varphi(x)$ may be estimated as $\Gamma \sim E_{\text{int}}/m_q$. Hence, for heavy mesons (e.g., for Υ particles), $\varphi(x)$ is rather narrow since $m_q \gg M \sim \Lambda_{\text{QCD}} \sim 300\,\text{MeV}$, where M

is a parameter characterizing interaction strength. On the other hand, taking $m_{u,d} \lesssim 10\,\mathrm{MeV}$ for the quarks in the pion, we conclude that $\varphi_\pi(x)$ is very broad. Assuming a simple exponential model

$$\Psi(x, k_\perp) \sim \exp\left[-\frac{k_\perp^2 + m_q^2}{M^2 x(1-x)}\right] \tag{9}$$

for the light-front wave function gives

$$\varphi_\pi(x, \mu \sim M) \cong f_\pi \exp\left[-\frac{m_q^2}{M^2 x(1-x)}\right] . \tag{10}$$

In this case, the pion DA $\varphi_\pi(x)$ is close to f_π everywhere outside the regions $0 \leq x \lesssim m_q^2/M^2 \sim 10^{-3}$ and $0 \leq 1-x \lesssim m_q^2/M^2 \sim 10^{-3}$. In these regions, $\varphi_\pi(x)$ vanishes rapidly.

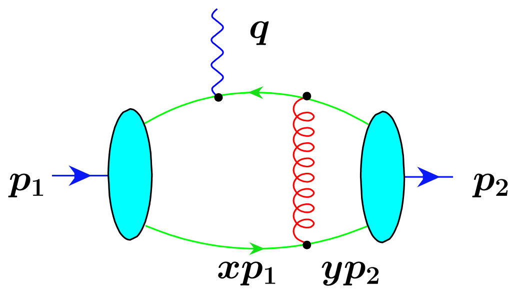

FIG. 1: One-gluon-exchange diagram for pion electromagnetic form factor in perturbative QCD.

Initially, the pion DA appeared in the perturbative QCD expression [1]

$$F_\pi^{\mathrm{as\,(pQCD)}}(Q^2) = \frac{8\pi\alpha_s}{9Q^2} \int_0^1 dx \int_0^1 dy \, \frac{\varphi_\pi(x)\, \varphi_\pi(y)}{xyQ^2} \tag{11}$$

for the asymptotics of the pion form factor calculated through a one-gluon-exchange diagram (see Fig.1). Here, xyQ^2 is the virtuality of the exchanged gluon. If one takes the flat pion DA $\varphi_\pi(x) = f_\pi$, both integrals in x and y logarithmically diverge, which means that pQCD factorization fails in this case. Evidently, the finite size $R \sim 1/M$ of the pion should provide a cut-off for the x, y integral, which suggests that xyQ^2 should be substituted by something like $xyQ^2 + \mathcal{O}(M^2)$, with the additional term having a meaning of the average of the squared transverse momentum of the quarks. One may also treat $\mathcal{O}(M^2)$ as an effective gluon mass squared. Then the x, y integrals are convergent, but the integral is dominated by the region where the nominal gluon virtuality is $\mathcal{O}(M^2)$. This means, first, that it is small, and what is even more important, that it is not growing with Q^2. For these reasons, the gluon-exchange line cannot be treated as a part of a perturbative short-distance subprocess, in which virtualities of all the lines should be parametrically $\mathcal{O}(Q^2)$. Hence, it should be absorbed into the nonperturbative part of the diagram, i.e. into the soft pion wave function. The pion form factor must be then calculated in some nonperturbative way. Such a calculation was accomplished within the QCD sum rule approach [16, 17], with the results close to experimental data.

The same QCD sum rule approach was used [6] to calculate ξ^2 and ξ^4 moments of the pion distribution amplitude $\phi_\pi(\xi)$ (which is the original DA $\varphi_\pi(x)$ written as a function of the relative variable $\xi \equiv x - (1-x)$ and divided by f_π). The value of $\langle \xi^2 \rangle$ is a quantitative measure of the width of the distribution amplitude. In particular, $\langle \xi^2 \rangle$ is zero for the infinitely narrow DA $\varphi_\pi(x) = f_\pi \delta(x - 1/2)$, it equals to $1/5$ for the asymptotic DA (8) and to $1/3$ for the flat $\varphi_\pi(x) = f_\pi$ DA. The calculation of Chernyak and Zhitnitsky (CZ) [6] gave the result larger than $1/3$, namely $\langle \xi^2 \rangle = 0.40$ for the "bare" value that was attributed to the normalization point $\mu^2 = 1.5\,\mathrm{GeV}^2$ and then renormalized to the reference scale $\mu^2 = 0.25\,\mathrm{GeV}^2$, which resulted in $\langle \xi^2 \rangle = 0.46$. Without touching a subtle point whether a perturbative evolution to such a low scale is justified, we can say that the CZ results clearly indicate that the pion DA is a wide function, and a generalized flat DA of $\phi_\pi(\xi) = a + 3(1-a)\xi^2$ type could have been used as a model fitting the values of $\langle \xi^2 \rangle$ and $\langle \xi^4 \rangle$ obtained from the CZ calculation. However, the fitting model

$$\phi_\pi^{\mathrm{CZ}}(\xi) = \frac{15}{4}\, \xi^2\, (1-\xi^2) \tag{12}$$

was constructed from the sum of two first terms of the Gegenbauer expansion (7), which has $x(1-x)$ (or $1-\xi^2$) as an overall factor, thus excluding all models with DA's that do not linearly vanish at the end-points.

An implicit assumption of the CZ calculation is that it is sufficient to take into account only the two lowest condensates $\langle GG\rangle$ and $\alpha_s\langle\bar{q}q\rangle^2$ in the operator product expansion (OPE) for the relevant two-point correlator. An alternative attitude [18] is that the quark condensate $\langle\bar{q}(0)q(0)\rangle$ is just the first term in Taylor expansion of the nonlocal condensate $\langle\bar{q}(0)q(z)\rangle \equiv \langle\bar{q}q\rangle\, f(z^2)$ that explicitly appears at the initial steps of OPE calculations. Modeling $f(z^2)$ is an attempt to include the tower of higher local condensates of $\langle\bar{q}(D^2)^n q\rangle$ type. The change from purely local approximation $f(z^2)=1$ to nonlocal condensates (NLC) with a smooth function $f(z^2)$ that rapidly decreases for large z^2 modifies the QCD sum rule results for the moments of the pion DA: they become smaller. In particular, the initial NLC calculation [18] gave $\langle\xi^2\rangle = 0.25$, and the model DA proposed in Refs. [18, 19] is

$$\phi_\pi^{\rm MR}(\xi) = \frac{8}{\pi}\sqrt{1-\xi^2}\ , \tag{13}$$

which is wider than the asymptotic DA, but narrower than the flat DA. The NLC method was elaborated in later papers, see Ref. [20] for a review. The problem of the NLC approach is that while it attempts to model the towers of $\langle\bar{q}(D^2)^n q\rangle$ condensates, the towers of $\langle\bar{q}G^n q\rangle$ condensates are neglected. Recently, Chernyak [21] gave a specific example in which the two towers exactly cancel each other. This means that NLC results may underestimate the value of $\langle\xi^2\rangle$, and cannot exclude a possibility of large $\langle\xi^2\rangle \gtrsim 1/3$ values for the second moment and $\langle\xi^4\rangle \gtrsim 1/5$ values for the fourth moment of the pion DA. On the other hand, it is quite possible that NLC argumentation is not completely wrong, and CZ results overestimate the values of $\langle\xi^2\rangle$ and $\langle\xi^4\rangle$. In particular, recent lattice calculations [7, 8] give $\langle\xi^2\rangle \approx 0.29$ and 0.27, respectively, at the scale $\mu^2 = 4\,{\rm GeV}^2$, which produces $\langle\xi^2\rangle \gtrsim 0.3$ at scales $\sim 1\,{\rm GeV}^2$, but definitely not $\langle\xi^2\rangle \gtrsim 0.4$.

A general comment is that converting the obtained values of $\langle\xi^2\rangle$ and $\langle\xi^4\rangle$ into models for the pion DA one should not restrict the models by the requirement that DA's must be given by a few first terms of the Gegenbauer expansion. There is no *a priori* principle justifying such a requirement: it is just an assumption which may or may not be true.

III. PHOTON-PION TRANSITION FORM FACTOR

The form factor $F_{\gamma^*\gamma^*\pi^0}(q_1^2, q_2^2)$ relating two (in general, virtual) photons with the lightest hadron, the pion, plays a special role in the studies of exclusive processes in quantum chromodynamics. When both photons are real, the form factor $F_{\gamma^*\gamma^*\pi^0}(0,0)$ determines the rate of the $\pi^0 \to \gamma\gamma$ decay, and its value at this point is deeply related to the axial anomaly [22]. At large photon virtualities, this form factor has the simplest structure analogous to that of the form factors of deep inelastic scattering. As a result, comparing pQCD predictions [23, 24, 25, 26, 27, 28] with experimental data, one can get information about the shape of the pion distribution amplitude $\varphi_\pi(x)$. Experimentally, $F_{\gamma^*\gamma^*\pi^0}(q_1^2, q_2^2)$ for small virtuality of one of the photons, $q_2^2 \approx 0$, was measured at e^+e^- colliders by CELLO [29], CLEO [30] and recently by BaBar [12] collaborations.

A. Perturbative QCD

The behavior of photon-pion transition form factor at large photon virtualities was studied [23, 24, 25] within perturbative QCD (pQCD) factorization approach for exclusive processes [1, 4, 23, 31]. Since only one hadron is involved, the $\gamma^*\gamma^*\pi^0$ form factor has the simplest structure for pQCD analysis, with the nonperturbative information about the pion accumulated in the pion distribution amplitude $\varphi_\pi(x)$. Another simplification is that the short-distance amplitude for $\gamma^*\gamma^* \to \pi^0$ transition is given, at the leading order, just by a single quark propagator. Theoretically, most clean situation is when both photon virtualities are large, but the experimental study of $F_{\gamma^*\gamma^*\pi^0}(Q_1^2, Q_2^2)$ in this regime through the $\gamma^*\gamma^* \to \pi^0$ process is very difficult due to very small cross section.

In the lowest order of perturbative QCD, the form factor for transition of *two* virtual photons with momenta q_1, q_2 into a neutral pion with momentum $p = q_1 + q_2$ is given by contribution of the handbag diagram (see Fig.2)

$$F_{\gamma^*\gamma^*\pi}^{\rm pQCD,LO}(q_1^2, q_2^2) = -\frac{\sqrt{2}}{3}\int_0^1 \frac{\varphi_\pi(x)}{xq_1^2 + (1-x)q_2^2}\,dx\ . \tag{14}$$

Introducing the asymmetry parameter ω through $q_1^2 = -Q^2(1+\omega)/2$ and $q_2^2 = -Q^2(1-\omega)/2$ gives

$$F_{\gamma^*\gamma^*\pi}^{\rm pQCD}(Q^2,\omega) = \frac{2\sqrt{2}}{3Q^2}\int_0^1 \frac{\varphi_\pi(x)}{1+\omega(2x-1)}\,dx\ \equiv \frac{\sqrt{2}f_\pi}{3\,Q^2}\,J(\omega)\ . \tag{15}$$

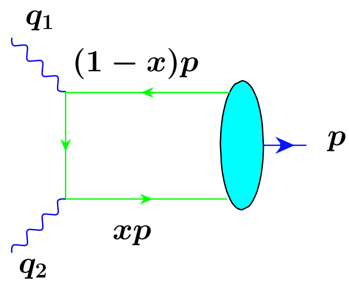

FIG. 2: Handbag diagram for photon-pion transition form factor.

Thus, if one would know the function $J(\omega)$, one could (in principle) obtain the pion DA $\varphi_\pi(x)$ by inverting the integral transform (15). However, as already mentioned, this kinematics is very difficult for experimental study. If one of the photons is real, i.e. $\omega = 1$, the leading-order pQCD prediction is

$$F^{\rm pQCD}_{\gamma^*\gamma\pi}(Q^2) = \frac{\sqrt{2}}{3Q^2}\int_0^1 \frac{\varphi_\pi(x)}{x}\,dx \;\equiv \frac{\sqrt{2}f_\pi}{3\,Q^2}\,J\ . \tag{16}$$

Information about the shape of the pion wave function is now accumulated in the factor J. It equals 2 for the infinitely narrow $\sim \delta(x-1/2)$ DA, for asymptotic DA (8) we have $J^{\rm as} = 3$, while CZ model (12) gives $J^{\rm CZ} = 5$. The intermediate distribution (13) produces $J^{\rm MR} = 4$. Thus, in addition to $\langle \xi^2\rangle$, we have another measure of the width of the pion DA, the value of J. Note, that for the DA's listed above, the ordering in J values is the same as the ordering in $\langle \xi^2\rangle$ values. However, the flat DA, for which $\langle \xi^2\rangle$ is smaller than that for the CZ model DA, generates infinite value for J, which is a consequence of the fact that it does not vanish at $x = 0$. This divergence of the integral for J formally means that the standard perturbative QCD factorization approach is not applicable for the flat DA case. But, since the divergence is only logarithmic, one may hope that some minimal fix, like a cut-off, might be sufficient. The question, of course, is whether there is a real need to use the flat DA to describe the data on the photon-pion transition form factor.

B. Logarithmic model

Recent data on $\gamma^*\gamma \to \pi^0$ form factor reported by BaBar collaboration in Ref. [12] are well fitted by the formula

$$Q^2\,F_{\gamma^*\gamma\pi^0}(Q^2) \cong \sqrt{2}f_\pi \left(\frac{Q^2}{10\,\text{GeV}^2}\right)^{0.25} \equiv \frac{\sqrt{2}f_\pi}{3}\,J^{\rm exp}(Q^2) \tag{17}$$

for the range $4\,\text{GeV}^2 < Q^2 < 40\,\text{GeV}^2$. The most startling observation is that $J^{\rm exp}(Q^2)$ does not show a tendency to flatten to some particular value. The specific $(Q^2)^\beta$ power-law parametrization of the growth is, of course, a matter of choice. In this region, $J^{\rm exp}(Q^2)$ is in fact very close to the logarithmic function

$$J^L(Q^2) = \ln\left(1+\frac{Q^2}{M^2}\right)\ , \tag{18}$$

if one takes $M^2 = 0.6\,\text{GeV}^2$, see Fig. 3. The two curves practically coincide for $Q^2 \gtrsim 15\,\text{GeV}^2$.

It is easy to notice that $J^L(Q^2)$ can be obtained if one uses the flat DA $\varphi_\pi(x) = f_\pi$ and changes $xQ^2 \to xQ^2 + M^2$ in the pQCD expression for the $\gamma^*\gamma \to \pi^0$ form factor:

$$J^L(Q^2) = Q^2\int_0^1 \frac{dx}{xQ^2+M^2}\ . \tag{19}$$

As discussed above, the idea to modify propagators $1/k^2 \to 1/(k^2+M^2)$ in integrals over the light-cone momentum fractions is rather old. The parameter M in such modifications is usually treated as the average transverse momentum of the propagating particle. However, the immediate observation is that the value $M = 0.77\,\text{GeV}$ is a little bit too large to be interpreted in such a way. Furthermore, the $1/xQ^2 \to 1/(xQ^2+M^2)$ modification is equivalent to bringing in, *before the integration over x,* a tower of $(M^2/xQ^2)^n$ power corrections, i.e., higher twists. But it is known [32]

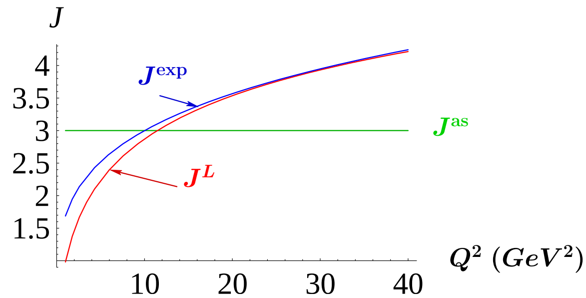

FIG. 3: Comparison of the function $J^{\rm exp}(Q^2)$ corresponding to the fit of BaBar data (blue online) and logarithmic model function $J^L(Q^2)$ (red online). The asymptotic pQCD prediction $J^{\rm as} = 3$ is also shown (green online).

that the handbag diagram, because of its simple singularity structure, cannot generate an infinite tower of power corrections. Indeed, the propagator of a massless quark in the coordinate representation is $\sim \not z (z^2)^{-2}$. Expanding the matrix element of the bilocal operator

$$\langle 0|\bar\psi(0)\gamma_5 \not z \psi(z)|p\rangle = \xi_2(zp)|_{z^2=0} + z^2\xi_4(zp)|_{z^2=0} + (z^2)^2\xi_6(zp)|_{z^2=0} + \dots \ , \tag{20}$$

we see that twist-6 and higher terms cancel the singularity of the propagator. Hence, there are just two terms in the OPE for the handbag contribution: twist-2 term that has $1/Q^2$ behavior and twist-4 term (corresponding to the $\bar\psi\gamma_5 \not z D^2\psi$ operator on the light cone) that gives $1/Q^4$ contribution. Operators with $(D^2)^{n\geq 2}$ do not contribute, and so there is no infinite tower of $(1/Q^2)^n$ terms.

C. Light-front formalism and Gaussian model

To investigate a possible mechanism capable of generating a cut-off at small x, let us write the $\gamma^*\gamma\pi^0$ form factor in the light-front formalism. The required expression was given in the classic paper [23] of Lepage and Brodsky on exclusive processes in QCD. Namely, the two-body (*i.e.*, $\bar q q$) contribution to the $\gamma^*\gamma\pi^0$ form factor is given by

$$(\epsilon_\perp \times q_\perp) F^{\bar q q}_{\gamma^*\gamma\pi^0}(Q^2) = \frac{1}{4\pi^3\sqrt{3}} \int_0^1 dx \int \frac{(\epsilon_\perp \times (xq_\perp + k_\perp))}{(xq_\perp + k_\perp)^2 - i\epsilon}\, \Psi(x, k_\perp)\, d^2k_\perp \, . \tag{21}$$

Here, $q_\perp$ is a two-dimensional vector in the transverse plane satisfying $q_\perp^2 = Q^2$, $\epsilon_\perp$ is a vector orthogonal to $q_\perp$ and also lying in the transverse plane [23], and the cross denotes the vector product. It can be shown that for the wave functions of $\Psi(x, k_\perp) = \psi(x, k_\perp^2)$ type we have [32]

$$F^{\bar q q}_{\gamma^*\gamma\pi^0}(Q^2) = \frac{1}{2\pi^2\sqrt{3}} \int_0^1 \frac{dx}{xQ^2} \int_0^{xQ} \psi(x, k_\perp^2)\, k_\perp dk_\perp \, . \tag{22}$$

Following [25], we take the Gaussian ansatz for the $k_\perp$-dependence of the light-front wave function, which we write in the form

$$\Psi^G(x, k_\perp) = \frac{4\pi^2 \varphi_\pi(x)}{x\bar x \sigma\sqrt{6}} \exp\left(-\frac{k_\perp^2}{2\sigma x \bar x}\right) , \tag{23}$$

where σ is the width parameter and $\varphi_\pi(x)$ is the desired pion distribution amplitude. The result for the form factor is then given by

$$F^G_{\gamma^*\gamma\pi^0}(Q^2) = \frac{\sqrt{2}}{3} \int_0^1 \frac{\varphi_\pi(x)}{xQ^2} \left[1 - \exp\left(-\frac{xQ^2}{2\bar x \sigma}\right)\right] dx \ . \tag{24}$$

It contains the $1/xQ^2$ pQCD contribution and a correction term which makes the integral convergent in the region of small x. An important observation is that the correction term in the integrand of Eq.(24) reflects the $k_\perp$ dependence of the nonperturbative pion wave function. In the Gaussian ansatz, this *integrand* term has an exponentially decreasing rather than a power behavior for large Q^2. This fact alone is sufficient to assert that it cannot be classified as a

higher-twist term. It comes from contributions invisible in the operator product expansion, which only sees the terms that have a powerlike behavior in $1/Q^2$ *before integration over* x. Representing this expression for the form factor as

$$F^{G}_{\gamma^*\gamma\pi^0}(Q^2) = \frac{\sqrt{2}f_\pi}{3}\, J^G(Q^2,\sigma)\ , \tag{25}$$

we find that, for the flat DA $\varphi_\pi(x) = f_\pi$, the function $J^G(Q^2,\sigma)$ has the following large-Q^2 asymptotic behavior:

$$J^G(Q^2,\sigma) = \ln\left(\frac{Q^2}{2\sigma}\right) + \gamma_E + \mathcal{O}(\sigma/Q^2)\ , \tag{26}$$

where γ_E is the Euler-Mascheroni constant. Comparing this result with the function $J^L(Q^2,M^2)$ (19) obtained through the M^2 modification of the pQCD $1/xQ^2$ propagator, we conclude that they have the same (up to $\mathcal{O}(1/Q^2)$ terms) asymptotic behavior if

$$\sigma = \frac{M^2}{2}\, e^{\gamma_E}\ , \tag{27}$$

which gives $\sigma = 0.53\,\text{GeV}^2$ for $M^2 = 0.6\ \text{GeV}^2$. In fact, plotting $J^L(Q^2, M^2 = 0.6\,\text{GeV}^2)$ and $J^G(Q^2,\sigma = 0.53\,\text{GeV}^2)$ together, we observe that these two functions practically coincide in the whole region $Q^2 > 1\,\text{GeV}^2$ we are interested in (see Fig. 4). Comparison of the model curve with BaBar experimental data is shown in Fig. 5.

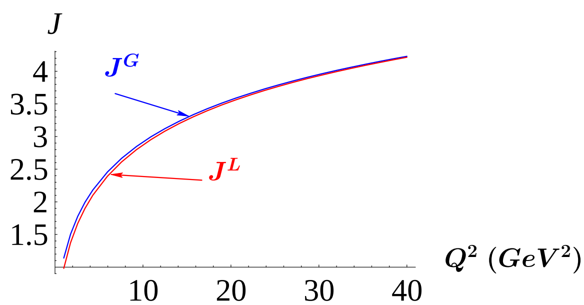

FIG. 4: Comparison of the logarithmic model $J^L(Q^2, M^2 = 0.6\,\text{GeV}^2)$ (red online) and Gaussian model $J^G(Q^2,\sigma = 0.53\,\text{GeV}^2)$ (blue online).

To check if the magnitude of σ is in a physically reasonable range, let us calculate the average transverse momentum for this Gaussian model. We have

$$\langle k_\perp^2(x)\rangle \equiv \int d^2k_\perp\, k_\perp^2\, \Psi(x,k_\perp) \left(\int d^2k_\perp\, dx\, \Psi(x,k_\perp)\right)^{-1} = 2\sigma\, x(1-x)\ , \tag{28}$$

and, hence,

$$\langle k_\perp^2\rangle \equiv \int_0^1 \langle k_\perp^2(x)\rangle\, dx = \frac{\sigma}{3}\ . \tag{29}$$

Thus, $\sqrt{\langle k_\perp^2\rangle} = 0.42\,\text{GeV}$, which is rather close to the folklore value of $300\,\text{MeV}$. One should also take into account that the wave function under consideration describes the valence two-quark Fock component of the pion, which is presumably smaller than other components.

Thus, the magnitude of the M^2-parameter of the logarithmic model is close to $3.3\,\langle k_\perp^2\rangle$ rather than to the value $\langle k_\perp^2\rangle$ expected from a naive substitution $xQ^2 \to xQ^2 + k_\perp^2$ in the quark propagator. As we explained, such a change has no theoretical grounds in the case of the handbag diagram. The justification of the *ad hoc* modification $xQ^2 \to xQ^2 + M^2$ used in our logarithmic model, as we have seen, is more complicated.

D. One-loop pQCD corrections

As already discussed, distribution amplitudes in general depend on the factorization scale μ, i.e. in principle one should always write: $\varphi(x,\mu)$. This dependence is induced by radiative corrections. The standard procedure in pQCD

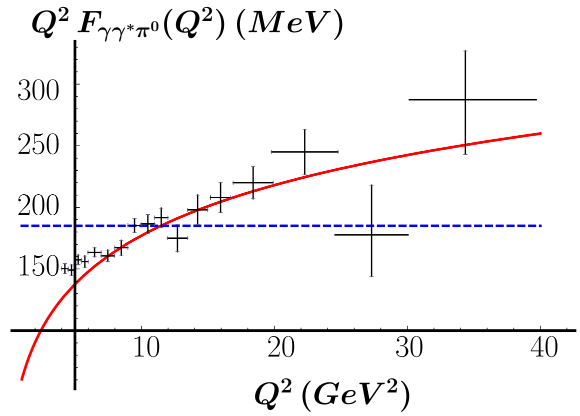

FIG. 5: Comparison of model curve (solid, red online) with BaBar experimental data. The asymptotic pQCD prediction $Q^2 F_{\gamma\gamma^*\pi^0}(Q^2) = \sqrt{2} f_\pi$ is also shown (dashed, blue online).

calculations involving pion DA is to start with an auxiliary quark-antiquark state in which the quarks are on shell and share the total momentum P in fractions xP and $(1-x)P$ according to the "bare" distribution amplitude $\varphi_0(x, m_q)$. Calculating radiative corrections for a specific process, e.g. for photon-pion transition form factor, one obtains logarithms $\ln(Q^2/m_q^2)$ accompanied by factors which may be converted into convolution of the lowest-order short-distance amplitude $T_0(x)$ with the evolution kernel $V(x,y)$ and the bare distribution amplitude $\varphi_0(y, m_q)$. Combining the evolution factor with bare DA, one obtains the expression in which $T_0(x)$ is multiplied by "evolved" distribution amplitude $\varphi(x, aQ)$, with a being some number, which is usually chosen in such a way as to minimize the size of that part of the corrections which was not absorbed into the renormalized (i.e. evolved) DA. One may also start with massless on-shell quarks, and use dimensional regularization to regularize mass singularities that result from $\ln(Q^2/m_q^2)$ terms for $m_q = 0$. Then the bare DA depends on the dimensional regularization scale μ, and one gets $\ln(Q^2/\mu^2)$ evolution logarithms calculating corrections to the amplitude of the short-distance subprocess.

The one-loop correction for the $\gamma^*\gamma \to \pi^0$ form factor was calculated in Refs.[26, 27, 28], with the result

$$\int_0^1 dx\, \frac{\varphi_\pi(x)}{xQ^2} \to \int_0^1 dx\, \frac{\varphi_\pi(x,\mu)}{xQ^2} \left\{ 1 + C_F \frac{\alpha_s}{2\pi} \left[\frac{1}{2} \ln^2 x - \frac{x \ln x}{2(1-x)} - \frac{9}{2} + \left(\frac{3}{2} + \ln x \right) \ln\left(\frac{Q^2}{\mu^2} \right) \right] \right\} \equiv f_\pi \frac{J(Q,\mu)}{Q^2} \, . \quad (30)$$

As advertised, the term containing the logarithm $\ln(Q^2/\mu^2)$ has the form of convolution

$$\frac{1}{xQ^2}\, C_F \frac{\alpha_s}{2\pi} \left(\frac{3}{2} + \ln x \right) = \int_0^1 \frac{1}{\xi Q^2}\, V(\xi, x)\, d\xi \quad (31)$$

of the lowest-order term $T_0(\xi, Q^2) = 1/\xi Q^2$ and the kernel

$$V(\xi, x) = \frac{\alpha_s}{2\pi}\, C_F \left[\frac{\xi}{x}\, \theta(\xi < x) \left(1 + \frac{1}{x - \xi} \right) + \frac{1-\xi}{1-x}\, \theta(\xi > x) \left(1 + \frac{1}{\xi - x} \right) \right]_+ \quad (32)$$

governing the evolution of the pion distribution amplitude. The "+"-operation is defined by

$$[F(\xi, x)]_+ = F(\xi, x) - \delta(\xi - x) \int_0^1 F(\zeta, x)\, d\zeta \, . \quad (33)$$

When the probing momentum Q is much larger than the initial normalization scale μ, one deals with large logarithm $\ln(Q^2/\mu^2)$. The latter can be eliminated by taking $\mu = Q$, and the expression is produced in which the *evolved* DA $\varphi_\pi(x, Q)$ is integrated with the remaining part of the correction. It is not guaranteed, however, that the resulting correction will be small, and the idea is to take $\mu = aQ$ with a chosen in such a way as maximally reduce the size of the α_s correction.

In the context of the present paper, we are interested in what happens when the bare DA is flat: $\varphi_0(x, \mu) = f_\pi$. Since in this case all integrals in (30) simply diverge, let us take a regularized version of the flat distribution amplitude,

namely the function

$$\varphi_r(x) = f_\pi \frac{\Gamma(2+2r)}{\Gamma^2(1+r)} x^r (1-x)^r \ , \tag{34}$$

with r being a very small parameter, say $r \lesssim 0.1$. Then Eq. (30) gives

$$J_r(Q,\mu) = \left(\frac{1}{r}+2\right)\left\{1 + \frac{\alpha_s}{3\pi}\left[\frac{2}{r^2} + \frac{\pi^2}{3} - 9 + \mathcal{O}(r) - \left(\frac{2}{r} - 3 + \frac{\pi^2}{3} r + \mathcal{O}(r^2)\right)\ln\left(\frac{Q^2}{\mu^2}\right)\right]\right\} \ . \tag{35}$$

It is clear that if we take $\mu = Q$, we will be left with a huge correction $\sim (2\alpha_s/3\pi)/r^2$, i.e. $\sim 60\,(\alpha_s/\pi)$ for $r = 0.1$. Since the coefficient in front of $\ln(Q^2/\mu^2)$ is dominated by $2/r$ term, while the μ-independent piece is dominated by its $2/r^2$ part, we can compensate the latter by taking $\ln(Q^2/\mu^2) = 1/r$. This corresponds to the choice

$$\mu^2 = Q^2\, e^{-1/r} \ . \tag{36}$$

Thus, if we take $r = 0.1$ to model the flat DA, the optimal choice for μ is something like $\mu^2 = 10^{-4}Q^2$. Even for the highest Q^2 reached in BaBar experiment, this gives $\mu^2 = 0.004\,\mathrm{GeV}^2$, a scale corresponding to distances much larger than the pion size. Evidently, we cannot evolve the pion DA down to such small momentum scales. The evolution must stop at some $\mu_0^2 \sim \Lambda^2_{\mathrm{QCD}}$. Thus, the flat pion DA becomes a DA at "low normalization point" $\mu = \mu_0 \sim \Lambda_{\mathrm{QCD}}$, below which there is no evolution. Moreover, as we have seen in the example above, the radiative corrections do not induce visible $\mathcal{O}(Q^2)$ additions to the renormalization parameter. Thus, in this "pQCD version" of the scenario with the flat DA, we deal simply with $\varphi_\pi(x)$. It does not evolve in the photon-pion transition amplitude, so there is no need to specify at which scale it is defined.

Furthermore, writing the square-bracketed term in Eq. (35) as $[A(r) - B(r)\ln(Q^2/\mu^2)]$, we can fine-tune the coefficient a by taking $a = \exp[-A(r)/B(r)]$ so as to completely eliminate the one-loop correction. Still, the resulting $\mu^2 = aQ^2$ will be very small, and there will be no evolution change in the shape of flat DA. In other words, in the pQCD version of flat DA scenario, there is no need to consider radiative corrections for the photon-pion transition form factor: they all are absorbed by the pion wave function.

The photon-pion transition form factor was investigated in Ref. [33] using large-N_c radial Regge model for resonances coupled to q_1 and q_2 photons. The results obtained in this way may be interpreted as a model with flat pion DA at low normalization point. In particular, $\sim \log Q^2$ behavior was obtained for $Q^2 F_{\gamma^*\gamma\pi^0}(Q^2)$ in the large-Q^2 region. The authors used the leading logarithm prescription $\phi_\pi(x) \to \phi_\pi(x,Q)$ to make comparisons with experiment, i.e. the pion DA in their approach is not exactly flat for large Q^2.

When the pQCD version of the scenario with flat pion DA is applied to pion electromagnetic form factor, the analysis of radiative corrections is very similar. The conclusion is that there is no need to consider the one-gluon-exchange diagram: the gluon line should be absorbed into the soft wave function. After that, only the soft contribution remains, and the form factor should be calculated nonperturbatively.

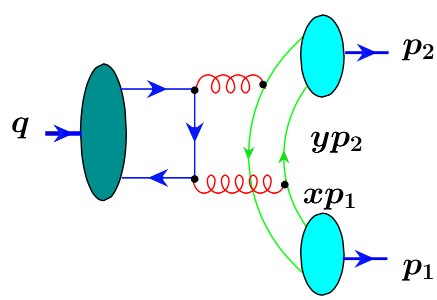

FIG. 6: Diagram for charmonium decay into two pions: the gluon lines cannot be absorbed into soft pion wave function

This does not mean that the flat DA scenario excludes the diagrams with gluon exchanges for all processes. Consider charmonium decays into two pions, $\chi_c \to \pi\pi$. The two gluons present in the lowest-order diagram cannot be absorbed into the pions' wave functions, so this diagram remains. In pQCD, it produces the same integrals of $\phi(\xi)/(1-\xi^2)$ type that diverge for flat DA. Thus, one should write a more detailed expression involving the $k_\perp$-dependent light-front wave functions for both pions, which is a challenging problem for future studies. The description of charmonium decays is a well-known success of CZ approach: if one uses pion DAs close to the asymptotic one, the theoretical results are well below the experimental data. In case of unmodified propagators, the flat scenario gives divergent

results for these amplitudes, while propagator modification brings them down to finite values. It is interesting to check if the resulting values are close to CZ ones.

One may argue that our logarithmic or Gaussian models for the lowest-order term are more or less equivalent to a simple cut-off of the x-integral at $x = M^2/Q^2$ value, which is essentially larger than $x \sim \exp[-1/r]$ values that are responsible for the dominant $1/r, 1/r^2, 1/r^3$ terms in the analysis above. If one simply imposes the cut-off at $x = M^2/Q^2$ in the pQCD expression (30), one would get powers of $\ln(Q^2/M^2)$ instead of of powers of $1/r$, and since $\ln(Q^2/M^2) \lesssim 4$ in our case, the asymptotically nonleading terms (especially (-9/2) contribution, see Eq.(30)) are essential. But it is not clear if a simple x cut-off in the pQCD expression is a correct prescription in the one-loop case. In particular, one may notice that the leading-order formula (22) of the light-front formalism can be formally written in terms of pion DA $\varphi_\pi(x,\mu)$ taken at $\mu = xQ$ (cf. Eq. (3)):

$$F^{\bar{q}q}_{\gamma^*\gamma\pi^0}(Q^2) = \frac{\sqrt{2}}{3}\int_0^1 \frac{dx}{xQ^2}\,\varphi_\pi(x,\mu = xQ)\,. \tag{37}$$

So, if the most important values are $x \sim M^2/Q^2$, then one should take $\mu \sim M^2/Q$ (which is $\mu \sim Q\,e^{-\ln(Q^2/M^2)}$, compare with (36)), i.e. again a very small value for large Q. However, to check if this reasoning extends to the one-loop case, one needs to calculate one-loop corrections in the light-front formalism keeping the $k_\perp$-dependence in the perturbative part and then convoluting the result with $k_\perp$-dependent nonperturbative wave function(s), which is a task going well beyond the scope of the present paper.

IV. SUMMARY

In this paper we discussed a scenario in which pion distribution amplitude is treated as a constant for all values of the light-cone momentum fraction x. We indicated that several approaches, in particular QCD sum rules and lattice gauge calculations give the values for the second moment $\langle \xi^2 \rangle$ of the pion distribution amplitude that are compatible with this proposal. We emphasized that the standard practice of building the model pion DAs as a sum of two or three lowest terms of the Gegenbauer expansion is just an assumption. Such an assumption, however, excludes flat DAs from the start. We calculated the photon-pion transition form factor using the light-front formula of Lepage and Brodsky and incorporating a $\bar{q}q$ wave function that gives flat pion DA and has a rapid (exponential, for definiteness) fall-off with respect to light-front energy combination $k_\perp^2/x(1-x)$. We demonstrated that the use of such a wave function is numerically equivalent to $1/xQ^2 \to 1/(xQ^2 + M^2)$ modification of the quark propagator, with the parameter M^2 being more than three times larger compared to the average square of the valence quark transverse momentum. The characteristic feature of our result is logarithmic $\sim \ln(Q^2/M^2 + 1)$ growth with Q^2 of the combination $Q^2 F_{\gamma*\gamma\pi^0}(Q^2)$. Such a growth is indicated by recent data of BaBar collaboration [12]. In this respect, it looks very important to check these results at other facilities, such as BELLE.

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